Curvature and the Perceptual Organization of Texture Flow

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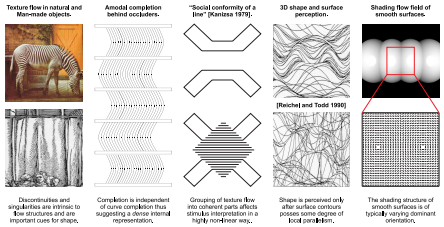
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1: Introduction

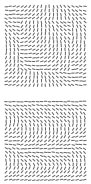
Texture flow (or orientation-defined texture) is a dense visual percept characterized by local (almost) parallelism and (typically varying) dominant local orientation. Its organization into coherent parts is fundamental to perceptual organization, shape interpretation, and shading analysis.



Which aspects of the *orientation content* influence orientation-based segmentation? Following texton theory and research into orientation gradients [e.g., Nothdurft 1985, Landy & Bergen 1991], current models for orientation-based segmentation depend *only* on the relationship between two *scalars* - the change of orientation *between* regions ($\Delta\theta_{between}$) and the change of orientation within regions ($\Delta\theta_{within}$) [e.g., Nothdurft 1991, Mussap & Levi 1999].

But:

☛ Orientation gradients, like any other gradient, are *vector* quantities whose direction (as opposed to magnitude) may have strong perceptual consequences.



The two patterns differ only in the direction of $\Delta\theta_{between}$. Its magnitude, as well as the value of $\Delta\theta_{within}$, are otherwise identical. Note the perceptual differences in the boundaries of the square.

☛ The same $\Delta\theta_{between}$ and $\Delta\theta_{within}$ can give rise to boundaries of very different perceptual saliency.

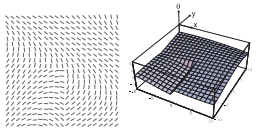


In this pattern $\Delta\theta_{between}$ is constant across the image and $\Delta\theta_{within}$ is constant along the boundaries (a square). ***And yet, the top edge is much more salient than the bottom one. Why?***

To understand what might cause these phenomena, a deeper understanding of the behavior of orientation in texture flows, and the parameters that govern it, is required. Geometrical analysis identifies two curvatures and psychophysical experimentation indicates that these curvatures are a critical factor in orientation-based segmentation, beyond $\Delta\theta_{between}$ and $\Delta\theta_{within}$!

2: Geometry and Representation

Any texture flow can be represented as a *scalar function* $\theta(x,y)$ over the image. Its 3D surface representation (where height=orientation) makes explicit the discontinuities (depicted as abrupt height change) and support the importance of the gradient. However, it provides no insight into the intrinsic geometry. For that we utilize the *frame field* representation [O'Neill 1966] which leads to the texture flow connection equation and its covariant derivatives:



$$\begin{aligned}\nabla_V E_T &= w_{12}(V)E_N \\ \nabla_V E_N &= -w_{12}(V)E_T\end{aligned}$$

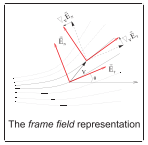
Natural basis leads to **two texture flow curvatures**

Tangential curvature : $\kappa_T \triangleq w_{12}(E_T)$

Normal curvature : $\kappa_N \triangleq w_{12}(E_N)$

which can be represented in terms of vector projection

$$\begin{aligned}\kappa_T &= \nabla\theta \cdot E_T = \nabla\theta \cdot (\cos\theta, \sin\theta) \\ \kappa_N &= \nabla\theta \cdot E_N = \nabla\theta \cdot (-\sin\theta, \cos\theta)\end{aligned} \quad (1)$$



The *frame field* representation

The frame field representation attaches an orthonormal frame (in red) to each point along the flow.

The covariant derivative ∇_V represents the rate of change orientation as the frame is translated on the texture flow along an arbitrary direction V.

Consequently, the tangential and normal curvatures are the covariant derivatives in the tangential and normal directions. They quantify how much the frame rotates as we move *parallel* or *perpendicular* to the flow, respectively.

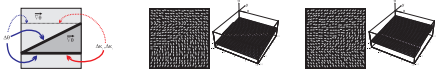
Consequences:

☛ ***The same orientation gradient can give rise to different curvatures!!***

☛ ***Could texture flow curvatures be involved in orientation-based segmentation? Special care is needed when testing such a hypothesis because unlike orientation gradients, texture flow curvatures cannot be simultaneously constant within a texture patch (however small) unless both are zero [Ben-Shahar & Zucker 2001] !!***

3: Methods

Stimuli: figure/ground-style texture flows with two possible configurations (for the figure)



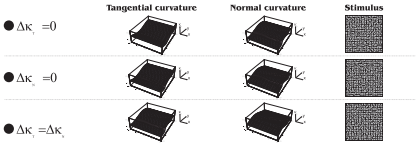
With...

☛ **constant** orientation gradient $\nabla\theta$ (=$\Delta\theta_{between}$), either {5, 10,15,20,25,30} degrees.

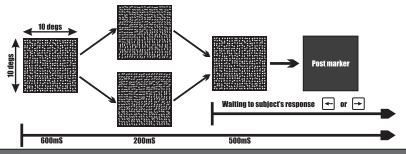
☛ **constant** orientation discontinuity $\Delta\theta$ (=$\Delta\theta_{within}$), either {5,10,15,20,...80,85,90} degrees.

☛ **constant** curvature discontinuities $\Delta\kappa_T$ and $\Delta\kappa_N$, either one of three possibilities

Note: Given $\nabla\theta$, $\Delta\theta$ and one $\Delta\kappa$, the other $\Delta\kappa$ is fully determined through Eq. (1).



Procedure: Two-Alternative Forced-Choice task. 20 presentations of each combination of $\nabla\theta, \Delta\theta$, and curvature discontinuities (i.e., 60 presentations of each combination of $\nabla\theta, \Delta\theta$) . Total of 6480 trials in six sessions of 1080 trials each.



4: Results

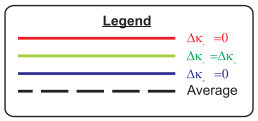
Ignoring the curvature dimension, performance is comparable to previous findings. Accuracy decreases with larger $\nabla\theta$ and smaller $\Delta\theta$.

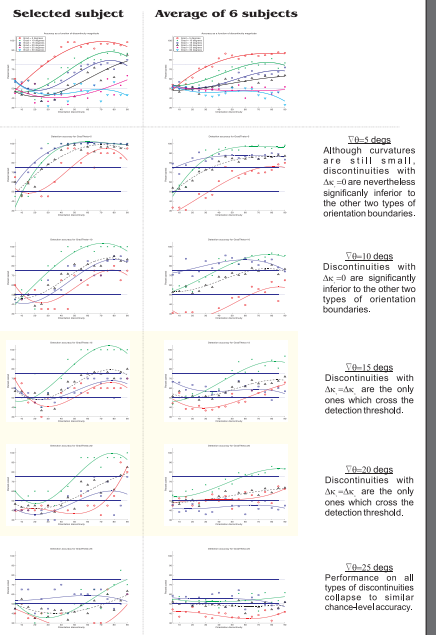
However, significant differences are revealed within each combination of $\nabla\theta$ and $\Delta\theta$ when curvature discontinuities are taken into account.

For small and intermediate $\nabla\theta$ values, a discontinuity with $\Delta\kappa = 0$ significantly reduced the "pop-out" effect and lowers segmentation performance.

More importantly...a discontinuity with $\Delta\kappa_T = \Delta\kappa_N$ typically produces the strongest perceptual effect. In particular, with intermediate-valued gradients ($10<\nabla\theta<25$), such discontinuities are the *only ones* , on average, to result in reliable segmentation (accuracy above 75%).

For $\nabla\theta$ higher than 25 degrees (graphs omitted here), no type of discontinuity surpasses the others and all of them, on average, lie around chance level.





5: Implications

☛ **Contrary to existing models, performance in orientation-based segmentation depends on *more than* the relationship between the (scalar values of the) rate of change of orientation within regions ($\Delta\theta_{within}$) and between regions ($\Delta\theta_{between}$). Since these notions are fundamental to preattentive vision in general, our findings justify a reexamination of the role of features, and feature *gradients*, in preattentive vision.**

☛ **Performance in orientation-based segmentation depends on discontinuities in flow *curvatures*.**

☛ **Psychophysics suggests the existence of a representation for curvature(s) in the human visual system. In particular, these experiments imply, for the first time, that the human visual system maintains a representation for a *normal* curvature, which demands an extension of current ''association field'' models dominated by curve integration [e.g., Field, Hayes, & Hess, 1993].**

☛ **Since flow curvatures are basic geometrical properties, an inquiry into the role of intrinsic geometry of other perceptual features (e.g., shading, color, motion) may be in place.**

☛ *Epilogue:* differences in curvature discontinuities explain the asymmetry in the saliency of the top and bottom edges of the Nothdurft square in the Introduction!!.

